UNIT - III BASEBAND TRANSMISSION & RECEPTION

Intersymbol Interference (ISI). Dec: 05, 17 (8) May: 09, 10, 18 (10, 16)

Describe the baseband txn system with neat diagram.

Baseband Transmission of Binary Data.

→ Binary data can be transmitted in baseband or passband.

→ Baseband txn → no modulation of high frequency carrier.

→ Passband txn → binary data modulates the carrier & the txn is takes place over the channel.

→ Baseband system for txn of digital data → PAM [Pulse Amplitude Modulation]

↳ amplitude of the pulses varies in discrete manner according to the i/p binary data.

→ PAM having only 2 amplitude levels → binary 1, binary 0

→ Successive binary digits combined into symbol(s)

→ PAM signals have multiple amplitude levels corresponding to these symbols.

→ Signals are transmitted without any modulation over the channel in baseband txn.

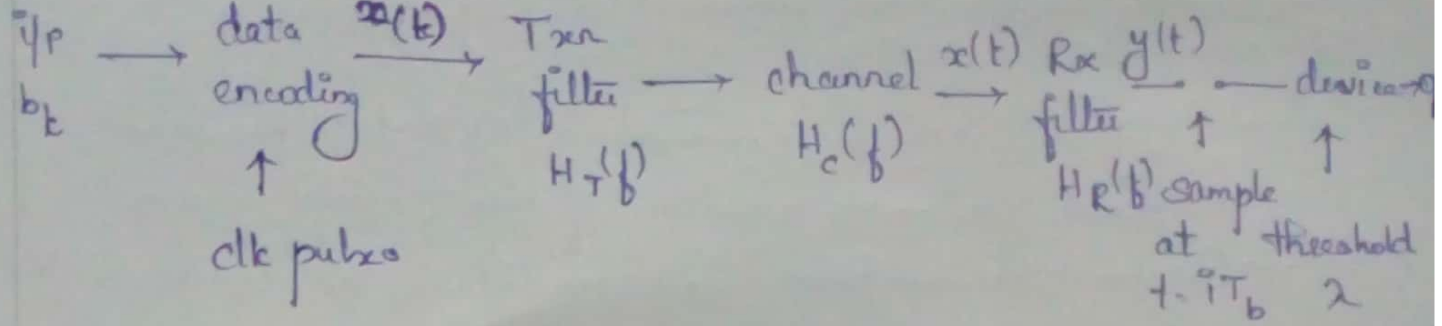
→ Binary data b_k is applied to the data encoder that generates pulse waveform $x(t)$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \, g(t - kT_b). \longrightarrow (1)$$

T_b → bit duration

g → pulse shaping

$$a_k = \begin{cases} +a, & b_k = 1 \\ -a, & b_k = 0 \end{cases} \longrightarrow (2)$$



Baseband binary data txn system.

→ $x(t)$ is passed through the txn filter. Txn filter combines all the txn circuits & signals. Combined transfer fn of the txn filter is $H_T(f)$. Passed through the channel with transfer fn $H_c(f)$. Channel delivers the signal to the rx filter consists of all necessary rx circuits & system having combined transfer fn $H_R(f)$. Output is $y(t)$ which is the replica of the txn signal $x(t)$.

→ $y(t)$ sampled synchronously with the Tx & sampling instants $t = iT_b$ that are synchronous to the clk pulses at Tx. Sampled signal $y(t_i)$ is given to the decision device that compares with the threshold λ.

$$\text{decision} \begin{cases} y(t_i) > \lambda \longrightarrow \text{sym } 1 \\ y(t_i) \le \lambda \longrightarrow \text{sym } 0 \end{cases}$$

ISI problem.

O/p of the system $y(t) = \underbrace{\mu}_{\text{scaling factor}} \sum_{k=-\infty}^{\infty} a_k \underbrace{p(t - kT_b)}_{\text{pulse shape}} \longrightarrow (3)$

$a_k g(t)$ → signal applied to the i/p of cascade of txn filter, channel and rx filter. o/p → $\mu a_k p(t)$.

Let the fourier transform of $g(t)$ be $G(f)$ and $p(t)$ be $P(f)$ then in frequency domain

$$\mu a_k P(f) = H(f) a_k G(f) \longrightarrow (4)$$

$$H(f) = H_T(f) H_c(f) H_R(f) \xrightarrow{(5)}$$ combined transfer fn of Txn filter, channel, Rx filter

(4) written as

$$\mu P(f) = H_T(f) H_c(f) H_R(f) G(f) \longrightarrow (6)$$

Rx filter o/p is sampled at $t_i = iT_b$

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} a_k \, p(iT_b - kT_b)$$

$$= \mu \sum_{k=-\infty}^{\infty} a_k \, p[(i-k)T_b] \longrightarrow (7)$$

Above eqn can be rewritten as

$$y(t_i) = \mu A_i \, p(0) + \mu \sum_{k=-\infty}^{\infty} a_k \, p[(i-k)T_b] \longrightarrow (8)$$

$$= \underbrace{\mu A_i}_{\text{o/p value } y(t_i) \text{ at } i=k.} + \underbrace{\mu \sum_{k=-\infty}^{\infty} a_k \, p[(i-k)T_b]}_{\text{residual effect of all bits}} \quad i=k$$

The presence of o/p due to other bits interfere with the o/p of the required bit is called ISI. Not consider the effect of channel noise actually channel noise & ISI both interfere the txn signal.

ISI absent $\longrightarrow y(t_i) = \mu A_i$

$\hookrightarrow$ decoded correctly hence eliminates the 2^{nd} term

Nyquist Criterion for Distortionless Txn. Dec - 04, 05, 06, 09, 16, 17, 18
May - 4, 5, 9, 10, 11, 12, 13, 14, 17, 18 (10, 8) (8)

Obtain an expression for Nyquist criterion for distortionless baseband txn for zero ISI.

The transfer fn of the channel and the transmitted pulse shape are specified. To determine the transfer fn of the txn & rx filter for that reconstruct the txn data sequence b_k. The rx does this by extracting & then decoding the corresponding sequence of weights a_k from the o/p $y(t)$.

$y(t)$ is determined by a_k & the rx pulse $p(t)$. Decoding requires that the weighted pulse contribution $a_k\, p(iT_b - kT_b)$ for $k = i$ be free from ISI due to the overlapping of all weighted pulse contributions represented by $k \neq i$. Rx pulse $p(t)$ is

$$p(iT_b - kT_b) = \begin{cases} 1 & i = k \\ 0 & i \neq k \end{cases} \longrightarrow (1)$$

If $p(t)$ satisfies the cond of the above eqn then the rx o/p is

$$y(t_i) = \mu a_i \longrightarrow \text{Zero ISI}$$

→ perfect reception in the absence of noise

ie $P(mT - kT) = \begin{cases} 1 & k = m \\ 0 & k \neq m \end{cases}$

Sampling in the Time domain produces periodicity in the frequency domain.

$$P_\delta(f) = R_b \sum_{n=-\infty}^{\infty} P(f - nT_b) \longrightarrow (2)$$

→ bit rate $R_b = 1/T_b$.

FT. fourier transform of an infinite periodic sequence of delta fn

Weighted of the sample value of p(t)

$$P_\delta(f) = \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \left[P(mT_b)\, \delta(t - mT_b) \right] \exp(-j2\pi f t)\, dt \longrightarrow (3)$$

→ integer m = i − k

i = k → m = 0 i ≠ k → m ≠ 0

according to the cond of eqn (1) integral of p(t)

$$P_\delta(f) = \int_{-\infty}^{\infty} P(0)\,\delta(t) \exp(-j2\pi f t)\, dt = P(0) \longrightarrow (4)$$

$P(0) = 1$ → Normalization → (2) & (4) cond for zero ISI is satisfied if

$$\sum_{n=-\infty}^{\infty} P(f - nR_b) = T_b \longrightarrow (5)$$

(5) eqn constitutes Nyquist Criterion for distortionless baseband txn in the absence of noise. Provides a method for constructing band-limited fn to overcome the effects of ISI. Depends on sampling the rx signal at midpoints of the signalling intervals.

<u>Ideal Soln.</u>

A frequency fn p(f) occupying the narrow band that satisfies equ (5) permitting only non zero component of f in $-B_0$ to $+B_0$ where B_0 → half the bit rate.

$$B_0 = R_b/2$$

$$P(f) = \frac{1}{2B_0} \operatorname{rect}\left(\frac{f}{2B_0}\right) \longrightarrow (6)$$

→ No frequencies of absolute value exceeding half the bit rate are needed. One signal waveform produces zero ISI is defined by the sinc fn.

$$p(t) = \operatorname{sinc}(2B_0 t) \longrightarrow (7)$$

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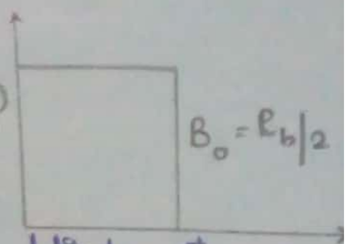
The fn $p(t)$ can be regarded as the impulse response of an ideal LPF with passband amplitude response $1/2B_0$ & B/W B_0. The fn $p(t)$ has its peak value at the origin & goes through 0 at integer multiples of $1/B_0$ the bit duration T_b.



If $y(t)$ is sampled at the time instants $t=0$ then the pulses defined by $\mu P(t - iT_b)$ with arbitary amplitude μ and index i will not interfere with each other.

Pulse shape $p(t)$ achieves B/W that solves the problem of zero ISI with min B/W.

2 practical difficulties

→ Requires the amp char of $p(f)$ be flat from $-B_0$ to $+B_0$ and 0 → unrealizable becoz of the abrupt transitions.

→ fn $p(t)$ decreases as $\frac{1}{|t|}$ for large $|t|$ resulting in a slow rate of decay → discontinuity of $p(f)$ becoz no error in sampling.

absence of noise

$$y(\Delta t) = \mu \sum_k a_k P(\Delta t - kT_b) \longrightarrow (8)$$

$$= \mu \sum_k a_k \frac{\sin\left[2\pi B_0 (\Delta t - kT_b)\right]}{2\pi B_0 (\Delta t - kT_b)}$$

$$y(\Delta t) = \mu \sum_k a_k \operatorname{sinc}(2B_0 \Delta t - k) \qquad 2B_0 T_b = 1$$

(9)

defines the desired symbol & series represents the ISI caused by the timing error Δt in sampling o/p $y(t)$.

Practical Solution.

To overcome the practical difficulties, the ideal solution extends the B/W from B_0 to an adjustable value between B_0 & $2B_0$.

$$P(f) + P(f-2B_0) + P(f+2B_0) = \frac{1}{2B_0} \qquad -B_0 \le f \le B_0 \longrightarrow (10)$$

The frequency characteristics consists of a flat portion & roll off portion has a sinusoidal form as follows

$$P(f) = \begin{cases} \dfrac{1}{2B_0} & |f| < f_1 \\ \dfrac{1}{4B_0}\left\{1 + \cos\left[\dfrac{\pi(|f|-f_1)}{2B_0 - 2f_1}\right]\right\} & f_1 \le |f| \le 2B_0 - f_1 \\ 0 & f \ge 2B_0 - f_1 \end{cases}$$

the frequency f_1 & bandwidth B_0 are related by

$$\alpha = 1 - \frac{f_1}{B_0}$$

$\alpha \longrightarrow$ roll off factor

$\alpha = 0 \longrightarrow f_1 = B_0$

$\downarrow$

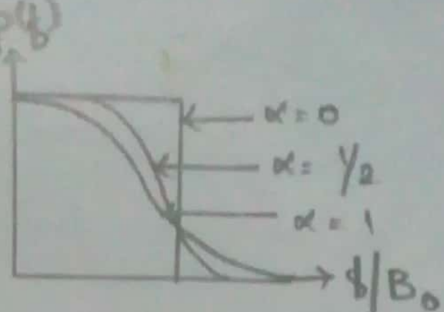
min B/W soln.

Frequency Response [P(f)].

P(f) normalized by multiplying it by $2B_0$, it has 3 values 0, 0.5, 1.

$\alpha = 0.5$ & $1 \longrightarrow$ roll off char of P(f) cuts gradually when compared with LPF char.

The fn $P(f)$ exhibits odd symmetry about the cut off frequency B_0 of ideal LPF.



freq response of α

Time Response $P(t)$.

$$P(t) = \operatorname{sinc}(2B_0 t)\,\frac{\cos(2\pi\alpha B_0 t)}{1-16\alpha^2 B_0^2 t^2}$$

Time response consists of product of 2 factors.

associated with the ideal filter

decreases as $1/t^2$ for large t so that the txn of binary waves using pulses is insensitive for sampling.

→ amt of ISI resulting from this timing error decreases as the roll off factor α is increased from 0 – 1. $P(t)$ is given by

$$P(t) = \frac{\operatorname{sinc}(4B_0 t)}{1-16B_0^2 t^2} \quad \because \alpha = 1.$$

Properties of Time Response.

1. At $t = \pm T_b/2 = \pm 1/4B_0 \rightarrow P(t) = 0.5 \Rightarrow$ Pulse width measured at half amplitude is exactly equal to bit duration T_b.
2. There are zero crossing at $t = \pm 3T_b/2, \pm 5T_b/2$ in addition to $\pm T_b, \pm 2T_b$.

→ 2 prop are used for generating timing signal from the rx signal for synchronization.

Ideal Case → $\alpha = 0$.

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Correlative Coding

→ based on ISI introduced into the txn signal is known

→ used to achieve max signaling rate of $2B_0$ bps in a BW of B_0 Hz.

Duobinary Signalling

Duo → doubling the txn capacity of a st. binary system is duobinary signaling to understand the basics of correlative coding.

sym 1 → pulse of amp +1v.

sym 0 → pulse of amp −1v.

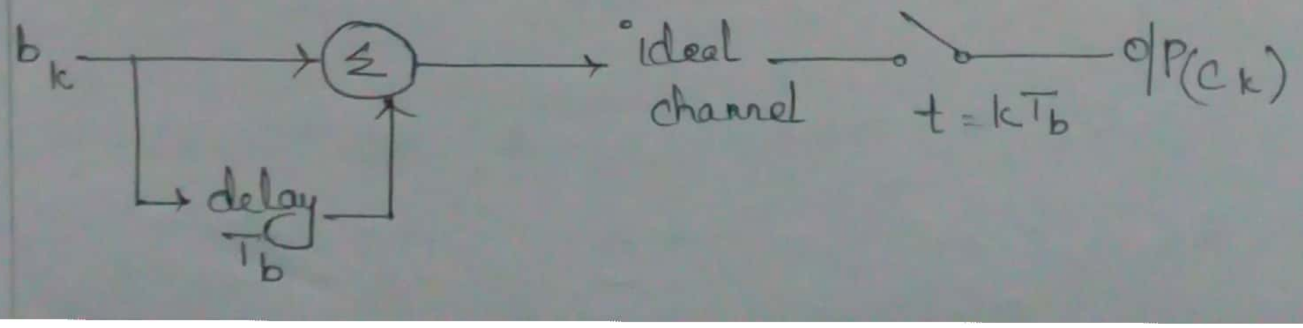
When seq is applied to duo binary encoder, converted into 3 o/p −2, 0, +2.

b_k is passed through a simple filter involving single delay element. C_k is the sum of the present binary digit b_k & previous value b_{k-1}

$$C_k = b_k + b_{k-1}$$

$b_k + b_{k-1}$ ↓ uncorrelated binary digits

C_k ↓ correlated digits.



Overall transfer f^n of the filter is

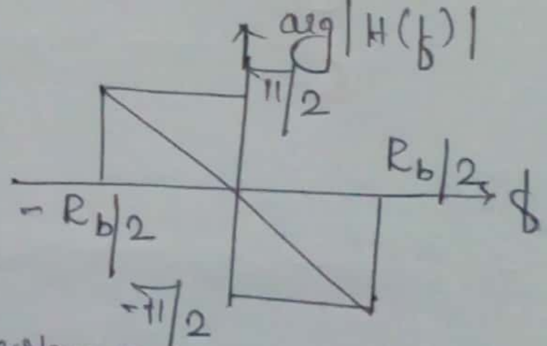
$$H(f) = 2\,H_c(f)\cos(\pi f T_b)\exp(-j\pi f T_b)$$

For ideal channel of B/W $B_0 = R_b/2$

$$H_c(f) = \begin{cases} 1 & |f| \leq R_b/2 \\ 0 & \text{other} \end{cases}$$

Overall frequency responce

$$H(f) = \begin{cases} 2\cos(\pi f T_b)\exp(-j\pi f T_b) & f \leq R_b/2 \\ 0 & \text{other} \end{cases}$$



$$h(t) = \frac{T_b^2 \sin(\pi t/T_b)}{\pi t(T_b - t)}$$

$\rightarrow$ Overall impulse filter response.

Estimate $\hat{b}_k = c_k - \hat{b}_{k-1}$

$c_k \rightarrow$ no error $\Rightarrow \hat{b}_{k-1} \rightarrow$ previous estimate of correct decision.

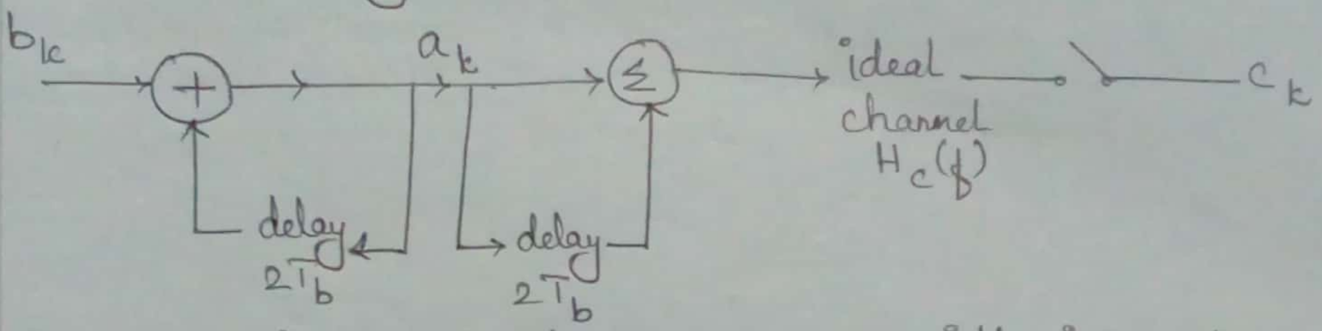
Technique of using stored estimate of previous symbol is called decision feedback.

$$a_k = b_k + a_{k-1} \text{ modulo } 2$$

$$\therefore\ c_k = a_k + a_{k-1}.$$

$$c_k = \begin{cases} \pm 2v & b_k \rightarrow \text{sym } 0 \\ 0v & b_k \rightarrow \text{sym } 1 \end{cases} \qquad b_k = \begin{cases} \text{sym } 0 & c_k > 1v \\ \text{sym } 1 & c_k < 1v \end{cases}$$

Modified duobinary tech.



Output of the modified duobinary conversion filter is related to the i/p seq

$$c_k = a_k - a_{k-2}$$

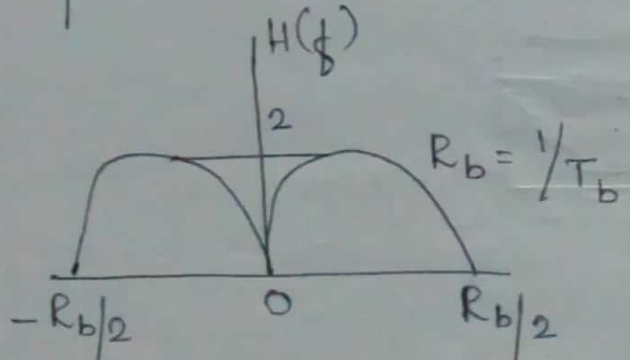
$c_k = 2, 0, -2 \longrightarrow a_k = \pm 1.$

$$H(f) = 2j\, H_c(f) \sin(2\pi f T_b) \exp(-j 2\pi f T_b)$$

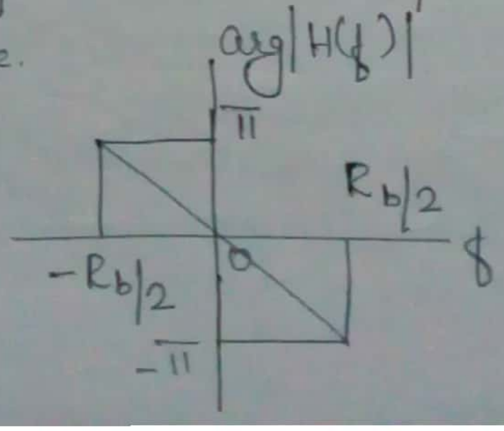
Overall freq response in the form of half cycle sine fn

$$H(f) = \begin{cases} 2j \sin(2\pi f T_b) \exp(-j 2\pi f T_b) & f \le R_b/2 \\ 0 & \text{other} \end{cases}$$

Amplitude Response.



Phase Response.



Phase response does not include constant 90° phase shift due to multiplying factor j. It has no dc component because many comm channels cannot transmit a dc component.

Impulse Response.

$$h(t) = \frac{2T_b^2 \sin(\pi t / T_b)}{\pi t (2T_b - t)}$$

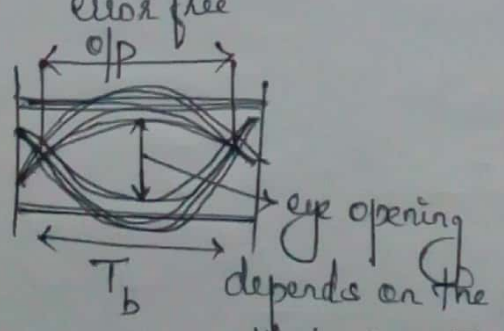
To eliminate the error probability $a_k = b_k + a_{k-2}$ mod-2.

Eye Pattern

→ to study the effect of ISI in baseband digital txn

→ when the sequence is txn over a baseband binary data txn system, o/p y(t) is a continuous time signal. ideally this is high/low depends on the data txn.

→ 'eye' given becoz it looks like an eye. this can also obtained on CRO if we apply y(t) to one of the i/p channels and apply trigger of $1/T_b$ which makes one sweep of beam equal to T_b.

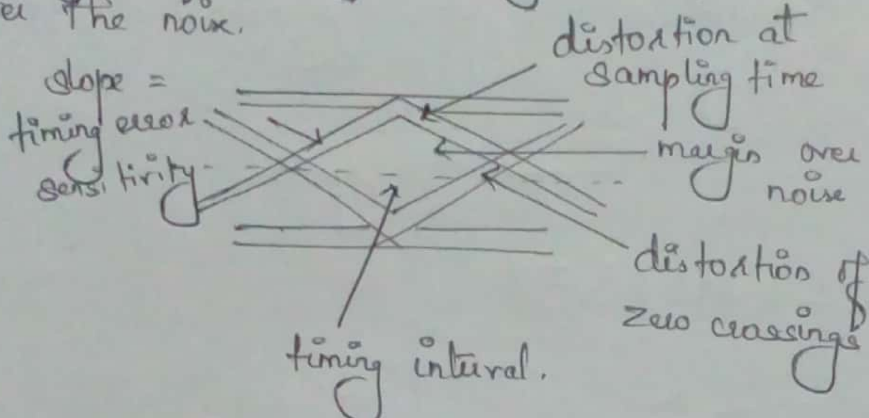


Eye Pattern

Performance of data txn system.

① Width of the eye opening defines the interval over which the rx wave can be sampled without error from ISI. Preferable to sample at the instant at which eye is open widest.

② Sensitivity of the system to timing error is determined by the rate of closure of eye as the sampling time is varied.
③ Height of the eye opening, at specified sampling time called margin over the noise.



Interpretation of the eye-pattern

→ as the effect of ISI increases, the eye opening reduces. if eye is closed completely, not possible to avoid errors in the o/p.
→ if there are M-levels then eye pattern contains (M-1) eye openings stacked vertically one upon the other.

Equalizing filters.

In the baseband transmission system, channel noise & ISI. Optimum linear receiver used for txn system

Zero forcing equalizer.

→ Optimum linear Rx realized with the help of zero forcing equalizer. Forces the ISI to 0.

o/p $y(t) = \int_{-\infty}^{\infty} c(\tau)\, x(t-\tau)\, d\tau$

$c(\tau)$ → impulse response of receive filter

$x(t)$ → i/p signal to receive filter.

$$x(t) = \sum_{k=-\infty}^{\infty} A_k\, q(t-kT_b) + w(t)$$

$w(t)$ → channel noise

A_k → symbol txn at $t = kT_b$

$$y(iT_b) = z_i + n_i$$

$n_i \longrightarrow$ noise component.

$z_i \longrightarrow$ signal component

error between the transmitted and received symbol will be

$$e_i = y(iT_b) - A_i$$

$A_i \longrightarrow$ txn symbol.

mean square error

$$MSE = \frac{1}{2} E(e_i^2)$$

$$\int_{-\infty}^{\infty} \left[R_q(t-\tau) + \frac{N_0}{2} \delta(t-\tau)\right] c(\tau)\, d\tau = q(-t)$$

$R_q(t,\tau) \longrightarrow$ temporal autocorrelation fn

$c(t) \longrightarrow$ impulse response of the zero forcing equalizer

Matched filter.

$\longrightarrow$ used for detection of signals in baseband & passband txn.

Req: SNR of the rx must be improved.

When SNR is max, signal must be checked at instant of bit period.

Error probability is min.

Called matched filter since its impulse response is matched to the shape of the i/p signal.

Decision Threshold $\longrightarrow$ assume that the rx signal is binary consider that the polar NRZ signal to represent binary 1's / 0's

binary 1 $\longrightarrow x_1(t) = +A$

binary 0 $\longrightarrow x_2(t) = -A$.

noisy i/p $x(t)+n(t) \longrightarrow$ Optimum filter $\longrightarrow$ sample $\longrightarrow$ o/p $r(t)$

$n(t) \rightarrow$ Noise $x(t) \rightarrow$ signal optimum filter i/p $= x(t) + n(t)$

$$\text{O/P from the Rx} = \begin{cases} x_{01}(T) + n_0(T) \\ x_{02}(T) + n_0(T) \end{cases}$$

Noise absent

$$\left.\begin{aligned} \text{O/P } r(T) &= x_{01}(T) \text{ if } x(t) = x_1(t) \\ r(T) &= x_{02}(T) \text{ if } x(t) = x_2(t) \end{aligned}\right\} \text{decision taken clearly.}$$

Noise present

$$\text{select} \begin{cases} x_1(t) \longrightarrow r(t) \text{ is closer to } x_{01}(T) \text{ than } x_{02}(T) \\ x_2(t) \longrightarrow r(T) \text{ is closer to } x_{02}(T) \text{ than } x_{02}(T) \end{cases}$$

$$\text{decision boundary} = \frac{x_{01}(T) + x_{02}(T)}{2}.$$

Error Cond.

$x_2(t) \longrightarrow$ txn but $x_{01}(T) > x_{02}(T)$. if noise $n_0(T)$ is +ve and larger in magnitude than the voltage difference

$\frac{1}{2}\left(x_{01}(T) + x_{02}(T)\right) - x_{02}(T)$ then incorrect decision will be taken

$$n_0(T) \geq \frac{x_{01}(T) - x_{02}(T)}{2}$$

$x_1(t) \longrightarrow$ txn but $x_{02}(T) > x_{01}(T)$, if noise $n_0(T)$ is −ve and less than $\frac{1}{2}\left(x_{01}(T) + x_{02}(T)\right) - x_{01}(T)$ then incorrect decision will be

$$n_0(T) \leq -\frac{x_{02}(T) - x_{01}(T)}{2}$$

Gen Error Prob.

① Gaussian Noise.

PDF of gaussian distributed fn

$$f_x(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x/m)^2/2\sigma^2}$$

m → mean value

$f_x(x)$ → pdf of rand variable x

σ → std deviation

evaluate pdf of white gaussian noise $x = n_0(t)$ & $m = 0$

$$f_x(n_0(t)) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(n_0(t))^2/2\sigma^2}$$

② Error Prob

$$P_e = \int_{\frac{x_{01}(T)-x_{02}(T)}{2}}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(n_0(t))^2/2\sigma^2}$$

$$= \frac{1}{2}\,\mathrm{erfc}\left[\frac{x_{01}(T)-x_{02}(T)}{2\sqrt{2}\,\sigma}\right]$$

Max Signal to Power Ratio.

① Noise Power

$$\sigma = (\text{mean square value} - \text{square of mean})^{1/2}$$

$$= \sqrt{\overline{x^2} - m_x^2}$$

$$\overline{x}^2 = \sigma^2 = \overline{n_0^2(t)} \qquad \because \overline{x^2} = \overline{n_0^2(t)}$$

(2) Signal Power.

$$P = x_0^2(T) / \sigma^2$$

(3) $x_0(T)$ interms of $H(f)$.

$$X_0(f) = H(f) X(f)$$

$$x_0(T) = IFT\{X_0(f)\}$$

$$= \int_{-\infty}^{\infty} H(f) X(f) e^{j2\pi f T} df$$

(4) σ^2 interms of $H(f)$.

$$P = x_0^2(T) / \sigma^2$$

$$= \left| H(f) X(f) e^{j2\pi f T} df \right|^2 \Big/ \int_{-\infty}^{\infty} |H(f)|^2 \frac{N_0}{2} df$$

(5) Schwarz's inequality.

$$P_{max} = \frac{2}{N_0} \int_{-\infty}^{\infty} | X(f)|^2 df \longrightarrow \text{max probab of error}$$

Min error probability $\rightarrow P_e = \frac{1}{2} \operatorname{erfc} \sqrt{E/N_0}$

Transfer fn of a matched filter $\longrightarrow H(f) = \frac{2k}{N_0} X^*(f) e^{-j2\pi f T}$

Impulse response $\longrightarrow h(t) = \frac{2k}{N_0} \{x_1(T-t) - x_2(T-t)\}$

Properties of matched filter.

(1) SNR of the matched filter depends on the ratio of signal energy to the psd of white noise.

$$P_{max} = 2E / N_0 .$$

(2) max signal component occurs at T = t & magnitude E ie energy of the signal x(t).

(3) o/p of the matched filter is proportional to a shifted version of the auto-correlation fn of i/p signal.