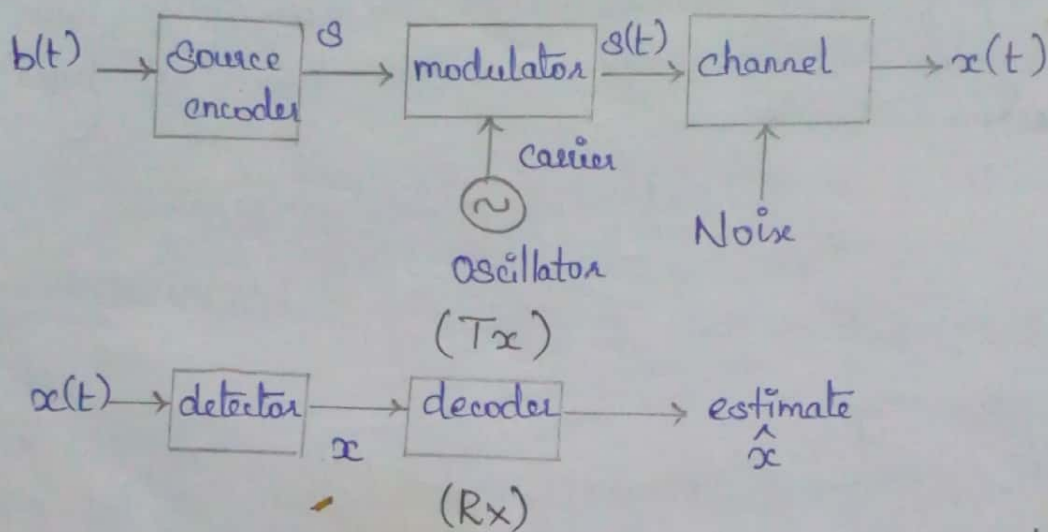


UNIT-4 DIGITAL MODULATION SCHEME

1) What is the need of digital modulations in digital communication? (or) State the advantages of digital carrier comm. over baseband digital transmission. Passband Transmission Model.



Model of passband transmission system.

- 1) Digital data \rightarrow is in the form of bits/bytes.
- 2) Source encoder \rightarrow produces the vector s_i .
- 3) Modulator \rightarrow constructs the modulated carrier signal $s_i(t)$.
- 4) Channel \rightarrow modulated signal is transmitted over the channel. channel noise is to be white Gaussian of 0 mean & $N_0/2$ psd.
- 5) Detector \rightarrow demodulates the received signal & estimates the signal vector.
- 6) Decoder \rightarrow obtains the estimate of symbol back from the signal.

Digital modulation Schemes.

- \rightarrow PSK (Phase Shift Keying) \rightarrow digital data modulates phase of the carrier
- \rightarrow FSK (Frequency Shift Keying) \rightarrow digital data modulates frequency of the carrier
- \rightarrow ASK (Amplitude Shift Keying) \rightarrow digital data modulates amplitude of the carrier.

Types of Passband Transmission.

- \rightarrow Coherent (Synchronous) detection \rightarrow the local carrier generated at the receiver is phase locked with the carrier at the transmitter.
- \rightarrow Non-coherent (Envelope) detection \rightarrow the receiver need not be phase locked at the transmitted carrier.

Advantages of Passband Transmission.

- Uses high carrier frequencies. Signals are transmitted over long distances with the help of cables, radio channels or fiber optic cables.
- Signals generated are analog in nature. Hence all types of analog channels are used for transmission.
- Bandwidth is conserved by this modulation techniques.
- Mainly suffers from ISI and crosstalk in baseband tx, these problems are absent in this modulation.
- Needs fiber optic / coaxial cables to carry the signals in baseband transmission whereas in passband signals are transmitted on radio channels.

Disadvantages of Passband Transmission.

- Complex system.
- Not suitable for short distance communication.

2) Explain Geometric Representation of Signals (or) How to represent the signals to be orthogonal to each other.
Geometric Representation of Signals.

Messages are transmitted over the carrier form a signal space that used to study the

- Probability of error
- Separation between the individual messages.

Vector space representation → characteristic of every digital modulation technique.

Let M be the no of energy signals form the i/p

$$\text{i.e. } y_i(t) = \{y_1(t) y_2(t) \dots y_M(t)\}$$

in terms of N no of orthonormal basis fn be

$$\phi_j(t) = \{\phi_1(t) \phi_2(t) \dots \phi_N(t)\}$$

the linear relationship between $y_i(t)$ & $\phi_j(t)$ be

$$y_i(t) = \sum_{j=1}^N y_{ij} \phi_j(t)$$

coefficients $y_{ij} = \int_0^T y_i(t) \phi_j(t) dt$

Orthogonal basis for $\int_0^T \phi_i(t) \phi_j(t) dt = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$

$\int_0^T \phi_i(t) \phi_i(t) dt = \int_0^T \phi_i^2(t) dt = 1 \quad (\because i=j)$

$\int_0^T \phi_i(t) \phi_j(t) dt = 0 \quad (\because i \neq j)$

ie. the basis for $\phi_1(t) \phi_2(t) \dots \phi_N(t)$ are orthogonal to each other over the interval $0 - T$.

Norm of the Vector.

Consider the vector $y_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iN} \end{bmatrix}$ and the product $\|y_i\|^2$ be $y_i^T y_i$

$\|y_i\|^2 = \sum_{j=1}^N y_{ij}^2 \rightarrow$ absolute value of a vector

basically length of the vector.

Relationship b/w signal energy & its vector

We know the signal energy be $E_i = \int_0^T y_i^2(t) dt$

$E_i = \int_0^T \left[\sum_{j=1}^N y_{ij} \phi_j(t) \right] \left[\sum_{k=1}^N y_{ik} \phi_k(t) \right] dt$

$= \sum_{j=1}^N \sum_{k=1}^N y_{ij} y_{ik} \int_0^T \phi_j(t) \phi_k(t) dt$

$\int_0^T \phi_j(t) \phi_k(t) dt = \begin{cases} 1 & j=k \\ 0 & j \neq k \end{cases}$

$\therefore E_i = \sum_{j=1}^N \sum_{k=1}^N y_{ij} y_{ik} \quad \begin{matrix} j=k \\ j \neq k \end{matrix}$

$$E_i = \sum_{j=1}^N y_i^2$$

$$i=j$$

$$= \|y_i\|^2 = \boxed{y_i^T y_i = E_i} \rightarrow \text{the energy signal is equal to the squared length of the signal vector.}$$

Euclidean distance.

Euclidean distance between y_i & y_k be

$$d_{ik} = \|y_i - y_k\|$$

Squared euclidean distance $\|y_i - y_k\|^2 = \sum_{i=1}^N (y_i - y_k)^2$

ie $E_i = \int_0^T y_i^2(t) dt = \|y_i\|^2$

$$\|y_i - y_k\|^2 = \int_0^T [y_i(t) - y_k(t)]^2 dt$$

angle θ_{ik} be $\cos \theta_{ik} = \frac{y_i^T y_k}{\|y_i\| \|y_k\|}$

$y_i^T y_k = 0 \rightarrow$ 2 vectors are orthogonal & perpendicular to each other.

3) Explain the Gram Schmidt Orthogonalization Procedure (or) Derive the expressions for the procedure of Gram Schmidt Orthogonalization. Gram Schmidt Orthogonalization Procedure.

\rightarrow a tool to obtain the orthonormal basis for $\phi_i(t)$.

① To derive $\phi_i(t)$

the set of M energy signals $y_1(t), y_2(t), \dots, y_M(t)$ for $y_i(t)$ the basis for be

$$\phi_1(t) = y_1(t) / \sqrt{E_1}$$

E_1 is the energy of $y_1(t)$ is $y_1(t) = \sqrt{E_1} \phi_1(t)$

$N=1 \rightarrow y_1(t) = y_{11} \phi_1(t)$

coeff $y_{21} = \int_0^T y_2(t) \phi_1(t) dt$

let a new fn be $g_2(t) = y_2(t) - y_{21} \phi_1(t)$
 the fn is orthogonal to $\phi_1(t)$ over the interval $0-T$ then

$$\phi_2(t) = g_2(t) / \sqrt{E_{g_2}}$$

here $\sqrt{E_{g_2}} = \int_0^T g_2^2(t) dt \rightarrow$ energy of $g_2(t)$.

② To prove $\phi_2(t)$ has unit energy

$$\begin{aligned} \text{Energy of } \phi_2(t) &= \int_0^T \phi_2^2(t) dt = \frac{1}{E_{g_2}} \int_0^T g_2^2(t) dt \\ &= \frac{1}{E_{g_2}} \cdot E_{g_2} = 1. \end{aligned}$$

Thus $\phi_2(t)$ has unit energy.

③ To prove $\phi_1(t)$ & $\phi_2(t)$ are orthogonal.

$$\begin{aligned} \text{Consider } \int_0^T \phi_1(t) \phi_2(t) dt &= \int_0^T \frac{y_1(t)}{\sqrt{E_1}} \cdot \frac{g_2(t)}{\sqrt{E_{g_2}}} dt \\ &= \frac{1}{\sqrt{E_1} \sqrt{E_{g_2}}} \int_0^T y_1(t) g_2(t) dt \\ &= \frac{1}{\sqrt{E_1} \sqrt{E_{g_2}}} \int_0^T y_1(t) [y_2(t) - y_{21} \phi_1(t)] dt \\ &= \frac{1}{\sqrt{E_1} \sqrt{E_{g_2}}} \left[\int_0^T y_1(t) y_2(t) dt - \int_0^T y_2(t) \phi_1(t) y_1(t) \phi_1(t) dt \right] \end{aligned}$$

$\therefore \int_0^T \phi_1(t) \phi_2(t) dt = 0 \rightarrow$ thus the 2 basis fn are orthogonal

④ Gen eqn for orthonormal basis fn

$$\phi_i(t) = g_i(t) / \sqrt{E_{g_i}} \quad i=1, 2, \dots, N \rightarrow \text{orthonormal basis fn}$$

$$\text{where } g_i(t) = y_i(t) - \sum_{j=1}^{i-1} y_{ij} \phi_j(t)$$

$$y_{ij} = \int_0^T y_i(t) \phi_j(t) dt \rightarrow \text{coeff}$$

4) Explain the maximum likelihood detection & obtain the decision rule. Prove that ML detector reduces to min. dist detector for white gaussian noise channel. Maximum likelihood decoding.

Let $x \rightarrow$ Observation vector, decision be $\hat{m} = m_i$ & avg prob of error

$$P_e(m_i, x) = P(m_i \text{ not sent} | x) = 1 - P(m_i \text{ sent} | x)$$

Optimum decision rule is $\hat{m} = m_i$ if $P(m_i \text{ sent} | x) \geq P(m_k \text{ sent} | x)$ for all $k \neq i$

Let R is N dim space of all vectors x and this region is divided M decision regions then the decision rule be $R_i = \{x \mid \ln[f_x(x/m_i)] \text{ is max for } k=i\}$.

$\rightarrow \ln[f_x(x/m_k)]$ is the likelihood for which results when symbol m_k is transmitted called max. likelihood and their detector called max. likelihood detector.

decision rule $\rightarrow x$ lies in R_i if $\|x - s_k\|$ is min for $k=i$.

Detection Process.

Let define $P(x_1) =$ Prob of x symbol x_1

$P(x_2) =$ Prob of x symbol x_2

$f_x(v/x_1) =$ pdf of x voltage v when x_1 is sent

$f_x(v/x_2) =$ pdf of x voltage v when x_2 is sent

$\lambda =$ decision threshold.

when $v > \lambda \rightarrow x_1$ selected and $v < \lambda \rightarrow x_2$ selected.

$$\text{Error Prob } P_e = P(x_1) \int_{v < \lambda} f_x(v/x_1) dv + P(x_2) \int_{v > \lambda} f_x(v/x_2) dv$$

$$\text{Sym 1} \rightarrow \int_{v > \lambda} f_x(v/x_1) dv + \int_{v < \lambda} f_x(v/x_1) dv = 1 \quad (\because \int_{-\infty}^{\infty} f_x(x) dx = 1)$$

$$\int_{v < \lambda} f_x(v/x_1) dv = 1 - \int_{v > \lambda} f_x(v/x_1) dv$$

$$v > \lambda \rightarrow \frac{P(v/x_1)}{P(v/x_2)} > \frac{P(x_2)}{P(x_1)} \rightarrow \text{max likelihood detector.}$$

$$\frac{P(v/x_1)}{P(v/x_2)} < \frac{P(x_2)}{P(x_1)} \rightarrow \text{max error probability.}$$

5) Explain ASK (or) OOK in detail.

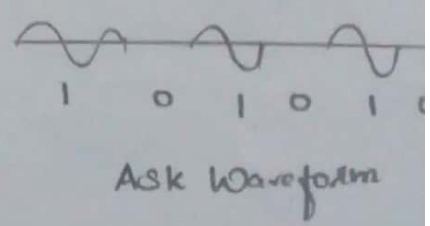
ASK (Amplitude Shift Keying / OOK On-Off Keying).

→ Simplest digital modulation technique.

→ there is only one unit energy carrier as it is switched ON/OFF depending upon the i/p.

ASK Waveform: $s(t) = \sqrt{2P_s} \cos(2\pi f_c t)$

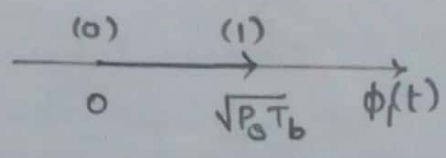
sym 0 → $s(t) = 0$ → no signal is transmitted.



sym 1 → $s(t) = 1$ → signal is transmitted.

Signal Space Diagram.

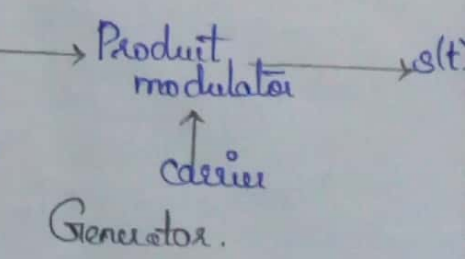
Symbol 1 → $s(t) = \sqrt{P_s T_b} \cdot \sqrt{2/T_b} \cos(2\pi f_c t)$
 $= \sqrt{P_s T_b} \phi_1(t)$



→ Only one carrier freq. Signal space has 2 pt, one at 0 and other at $\sqrt{P_s T_b}$. dist b/w 2 signal pt be $d = \sqrt{E_b}$.

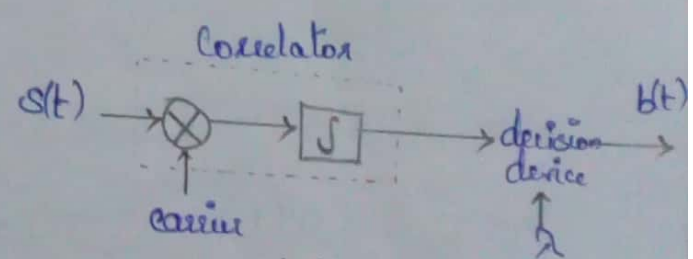
Coherent detector.

→ i/p binary sequence is applied to the product modulator that modulates the amplitude of the sine carrier.

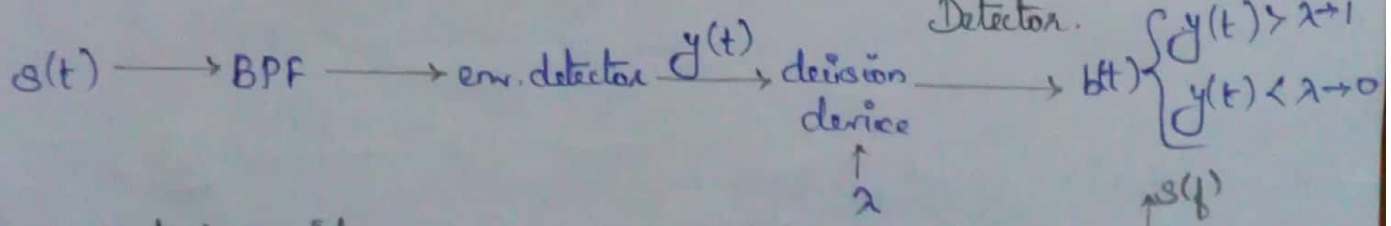


→ passes carrier → i/p 1
 blocks carrier → i/p 0.

→ decision $\begin{cases} \lambda > 1 \rightarrow 1 \\ \lambda < 1 \rightarrow 0 \end{cases}$

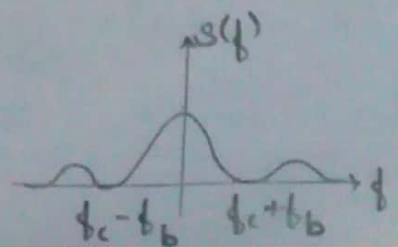


Non coherent Detection.



Power Spectral Density.

$S_{ASK}(f) = \frac{T_b}{4} \text{sinc}^2(f T_b) + \frac{1}{4} \delta(f)$



b) Derive an expression for the error probability of BFSK system. (or) Explain the operation of BFSK Tx and Rx (or) Describe the generation & detection of Binary Frequency Shift Keying (BFSK) of coherent binary FSK and explain P_e .

Principle:

In FSK, the frequency of the carrier is shifted according to the binary symbol. In this sym 0 & sym 1 are distinguished from each other by transmitting one of the 2 sine waves that differs in frequency.

$$S_i(t) = \begin{cases} \sqrt{2E_b/T_b} \cos(2\pi f_i t) & 0 \leq t \leq T_b \\ 0 & \text{otherwise} \end{cases}$$

$\therefore S_1(t) \rightarrow \text{sym 1}$ and $S_2(t) \rightarrow \text{sym 0}$

Energy:

Orthonormal basis for $\phi_j(t) = \begin{cases} \sqrt{2/T_b} \cos(2\pi f_j t) & 0 \leq t \leq T_b \\ 0 & \text{otherwise} \end{cases}$

coeff $S_{ij} = \int_0^{T_b} S_{ij}(t) \phi_j(t) dt = \begin{cases} \sqrt{E_b} & i=j \\ 0 & i \neq j \end{cases}$

Geometrical Representation:

\rightarrow characterized by having single space i.e. $N=2, M=2$.

2 msg pts are defined as $S_1 = \begin{pmatrix} \sqrt{E_b} \\ 0 \end{pmatrix}$ $S_2 = \begin{pmatrix} 0 \\ \sqrt{E_b} \end{pmatrix}$

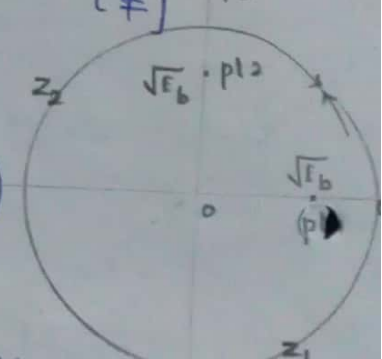
Observation vector has 2 elements $x_1 = \int_0^{T_b} x(t) \phi_1(t) dt$

$$x_2 = \int_0^{T_b} x(t) \phi_2(t) dt$$

Sym 1 $\rightarrow x(t) = S_1(t) + w(t) \rightarrow x$ falls inside z_1 when $x_1 > x_2$

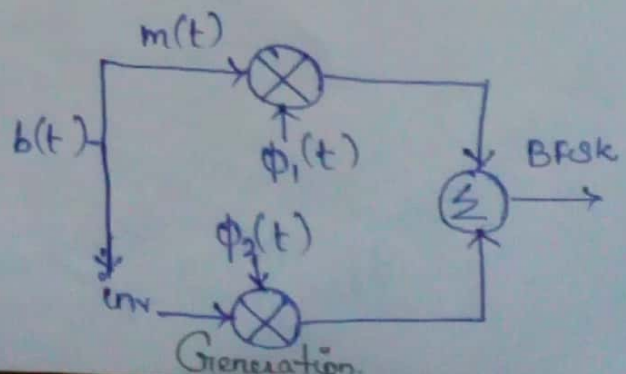
Sym 0 $\rightarrow x(t) = S_2(t) + w(t) \rightarrow x$ falls inside z_2 when $x_1 < x_2$

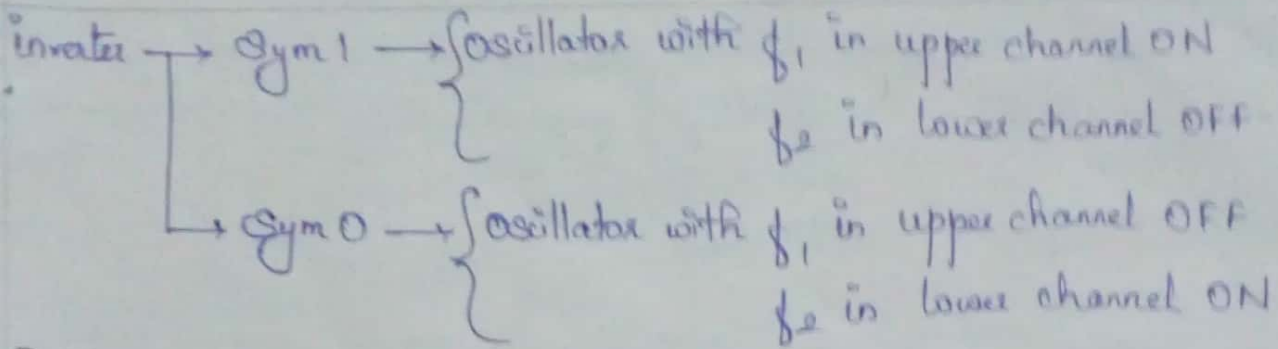
decision boundary $x_1 = x_2$ and distance $d = \sqrt{2E_b}$



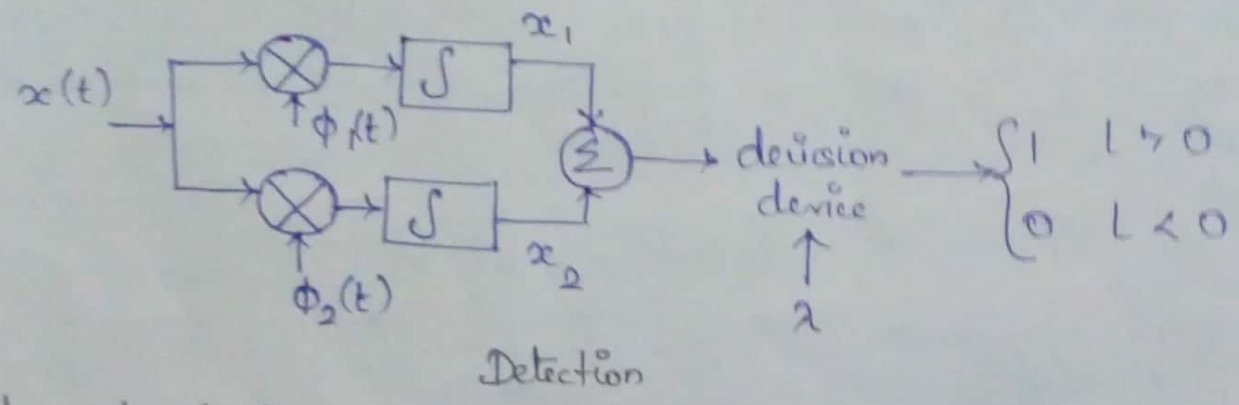
Coherent BFSK.

$\rightarrow b(t) \rightarrow \begin{cases} \sqrt{E_b} & \text{sym 1} \\ 0 & \text{sym 0} \end{cases}$

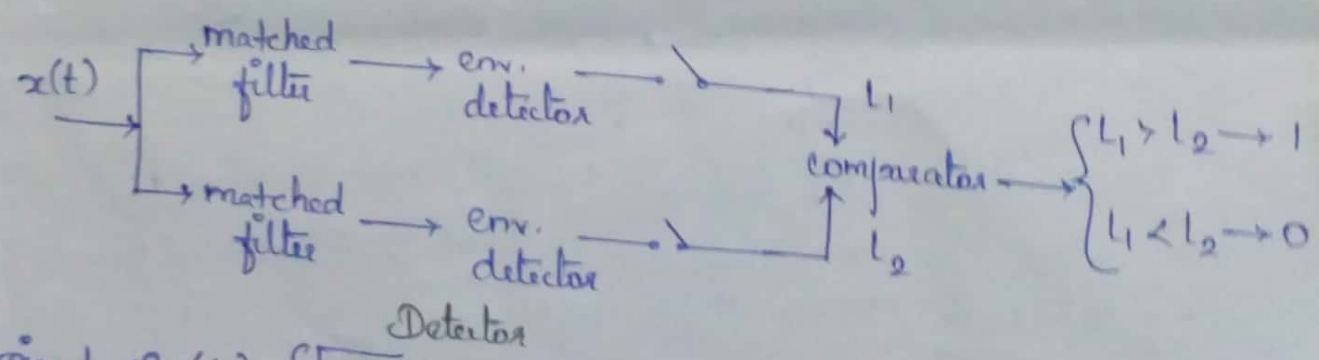




Receiver



Non-coherent Rx.



Tx signal $S_i(t) = \begin{cases} \sqrt{2E_b/T_b} \cos(2\pi f_i t) & 0 \leq t \leq T_b \\ 0 & \text{other} \end{cases}$

$f_1 \rightarrow \text{sym 1}$ and $f_2 \rightarrow \text{sym 0}$.

avg prob of error $P_e = \frac{1}{2} \exp(-E_b/2N_0)$

Bandwidth $BW = 4f_b$, Baud rate = f_b/N .

PSD of BFSK $S_B(f) = \frac{E_b}{2T_b} \left[\delta(f - \frac{1}{2T_b}) + \delta(f + \frac{1}{2T_b}) \right] + \frac{8E_b \cos^2(\pi f T_b)}{\pi^2 (4T_b f^2 - 1)^2}$

Error Prob $P_e = \frac{1}{2} \exp(-\frac{E_b}{2N_0})$

Advantages.

- Better noise immunity
- Simple

Disadvantages.

- Doubles the BW
- Poor error performance.

1) Explain the generation & detection of coherent BPSK and derive the PSD of BPSK (or) Derive the PSD of binary PSK signal (or) Calculate BER of BPSK (Binary Phase Shift Keying).

Principle.

In PSK, the phase of the carrier signal is varied according to the binary 'p'. The carrier phase is changed between 0° & 180° based on the binary 'p'. The pair of signals $S_1(t)$ & $S_2(t)$ used to represent sym 1 & 0.

$$\text{Sym 1} \rightarrow S_1(t) = \sqrt{2E_b/T_b} \cos(2\pi f_c t)$$

$$\text{Sym 0} \rightarrow S_2(t) = -\sqrt{2E_b/T_b} \cos(2\pi f_c t)$$

Energy.

→ There is only one basis fn of unit energy

$$\phi_1(t) = \sqrt{2/T_b} \cos(2\pi f_c t) \quad 0 \leq t \leq T_b.$$

$$\text{i.e. } S_1(t) = \sqrt{E_b} \phi_1(t) \text{ and } S_2(t) = -\sqrt{E_b} \phi_2(t).$$

Geometric Representation.

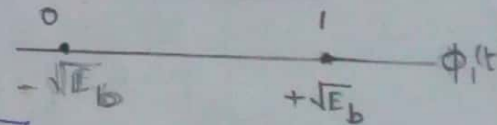
→ $N=1$ and $M=2$ the coordinates of msg pt be

$$S_{11} = \int_0^{T_b} S_1(t) \phi_1(t) dt = +\sqrt{E_b}$$

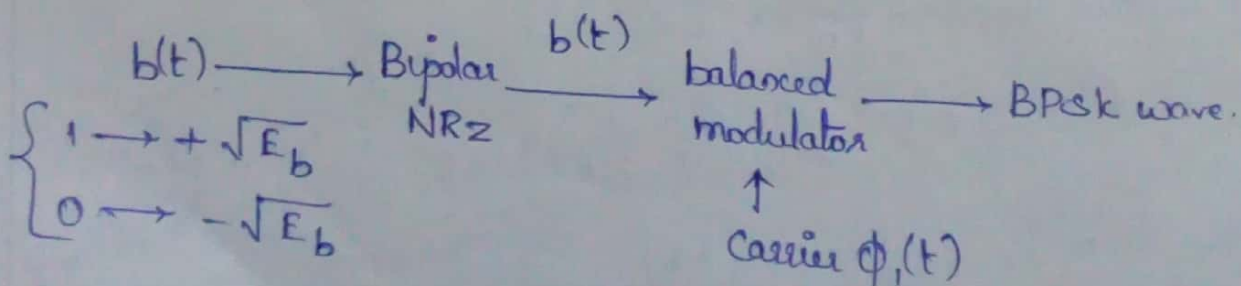
$$S_{21} = \int_0^{T_b} S_2(t) \phi_1(t) dt = -\sqrt{E_b}$$

distance

$$d = +\sqrt{E_b} - (-\sqrt{E_b}) = 2\sqrt{E_b}.$$

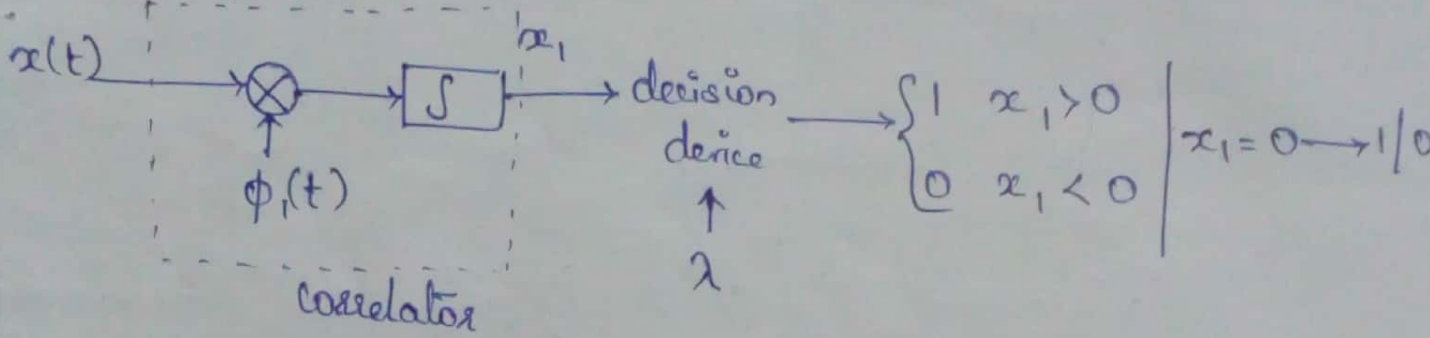


Generation.



Transmitter

Detection.



Receiver.

Bandwidth $BW = 2f_b \rightarrow f_b = 1/T_b$
 max BW is equal to twice the highest frequency contained in baseband signal.

Baudrate: Baudrate = f_b

PSD of BPSK be $S_B(f) = 2E_b \text{sinc}^2(fT_b)$

Error Prob. $P_e = \frac{1}{2} \text{erfc} \sqrt{\frac{E_b}{N_0}}$

Advantages.

- min error probability
- very good noise immunity
- lower BW

Disadvantages.

- Complex
- ambiguity in O/P signal.

8) Derive an expression of probability of error of QPSK signal (or) Explain QPSK TX & RX with suitable block dia and relevant expressions, QPSK (Quadrature Phase Shift Keying).

Goal:

the 2 important goals of the design of a digital communication system are

- Very low error probability
- Efficient utilization of BW.

Principle:

In QPSK, 2 successive bit in data sequences are grouped together to form symbols. This reduces the bit rate and reduces the channel BW. The phase of the carrier be $\pi/4, 3\pi/4, 5\pi/4$ & $7\pi/4$.

$$s_i(t) = \begin{cases} \frac{2E}{T} \cos \left[2\pi f_c t + (2i-1) \frac{\pi}{4} \right] & 0 \leq t \leq T \\ 0 & \text{other} \end{cases}$$

$$= \begin{cases} \sqrt{\frac{2E}{T}} \cos \left((2i-1) \frac{\pi}{4} \right) \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin \left((2i-1) \frac{\pi}{4} \right) \sin(2\pi f_c t) & 0 \leq t \leq T \\ 0 & \text{other} \end{cases}$$

Signal space representation.

→ 2 orthonormal basis for $\phi_1(t)$ & $\phi_2(t)$ are

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \quad \text{and} \quad \phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$$

→ 4 msg pt.

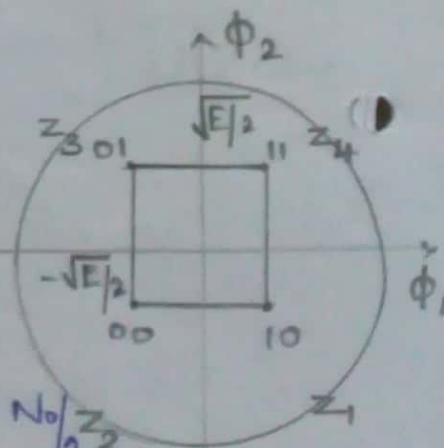
$$s_i = \begin{cases} \sqrt{E} \cos \left[(2i-1) \frac{\pi}{4} \right] \\ -\sqrt{E} \sin \left[(2i-1) \frac{\pi}{4} \right] \end{cases}$$

Geometrical Representation.

→ $N=2$, $M=4$.

rx signal $x(t) = s_i(t) + w(t)$ $0 \leq t \leq T$

↳ white gaussian noise of mean 0 & psd $N_0/2$

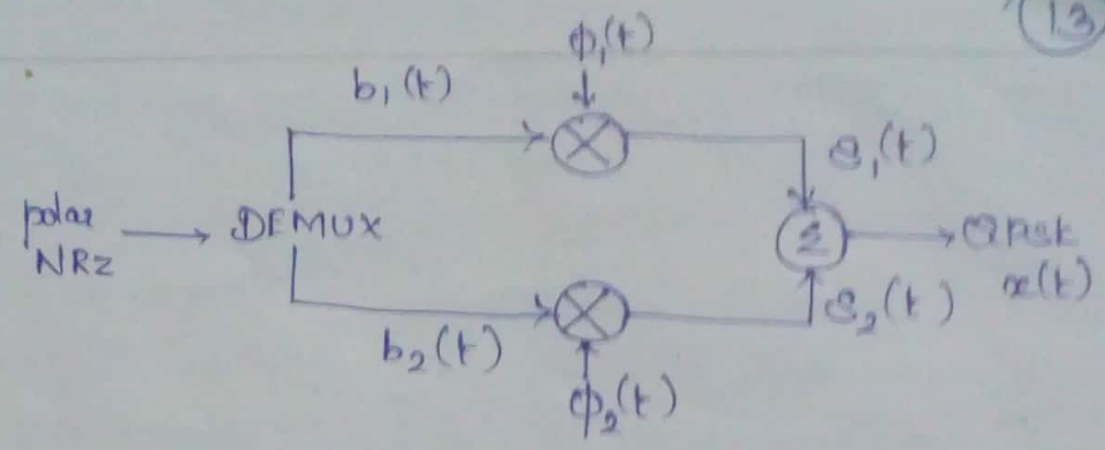


→ 2 elements $\Rightarrow x_1 = \int_0^T x(t) \phi_1(t) dt = \sqrt{E} \cos \left((2i-1) \frac{\pi}{4} \right) + w_1$

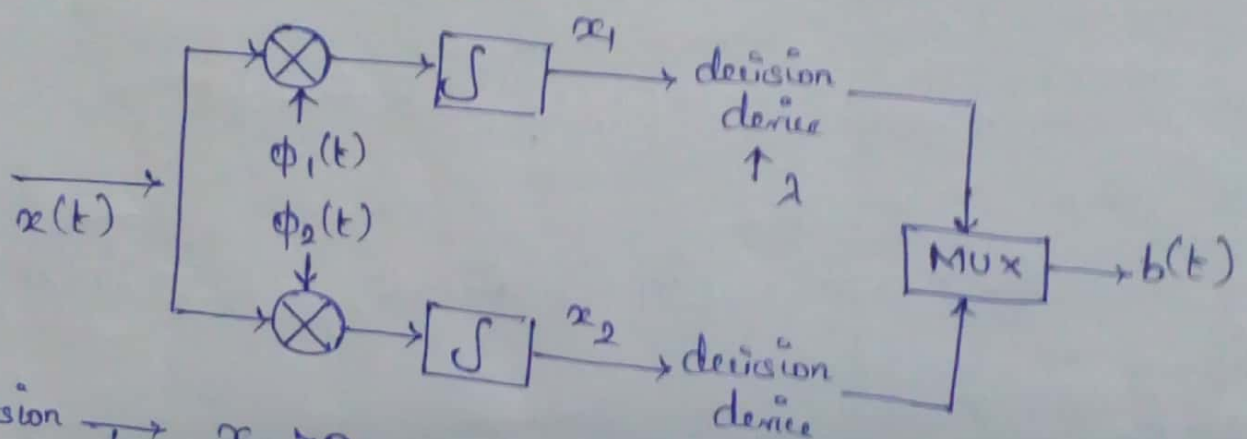
$$x_2 = \int_0^T x(t) \phi_2(t) dt = -\sqrt{E} \sin \left((2i-1) \frac{\pi}{4} \right) + w_2$$

Generation.

binary
 $b(t)$ seq
 $\left\{ \begin{array}{l} 1 \rightarrow \sqrt{E_b} \\ 0 \rightarrow -\sqrt{E_b} \end{array} \right.$



Detection.



decision

- $x_1 > 0 \rightarrow 1$
- $x_1 < 0 \rightarrow 0$
- $x_2 > 0 \rightarrow 1$
- $x_2 < 0 \rightarrow 0$

upper channel
 lower channel.

Bandwidth: $BW = f_b$

PSD \rightarrow Symbol shaping for $g(t) = \begin{cases} \sqrt{E_b/T} & 0 \leq t \leq T \\ 0 & \text{other} \end{cases}$
 \rightarrow inphase & quadrature component $S_B(f) = 4E_b \text{sinc}^2(2T_b f)$.

Error Prob.

$$P_e = \text{erfc} \sqrt{\frac{E_b}{N_0}}$$

Advantages.

- \rightarrow Reduced BW
- \rightarrow effective utilization of BW
- \rightarrow low prob of error

Disadvantages.

- \rightarrow interchannel interference is large
- \rightarrow Complex.

9) Explain QAM system with its Tx, Rx and signal space representation (or) Derive P_e for QAM.
QAM (Quadrature Amplitude Modulation)

Principle:

In all PSK, inphase and quadrature components of the modulated signal are interrelated in such a way that the envelope is constrained to remain constant. If this constraint is removed and the inphase & quadrature components are permitted to be independent such modulation scheme called QAM.

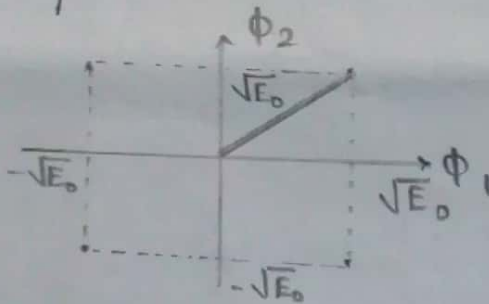
$$S_i(t) = \sqrt{\frac{2E_0}{T}} a_i \cos(2\pi f_c t) + \sqrt{\frac{2E_0}{T}} b_i \sin(2\pi f_c t) \quad 0 \leq t \leq T$$

S_i in terms of basis fn

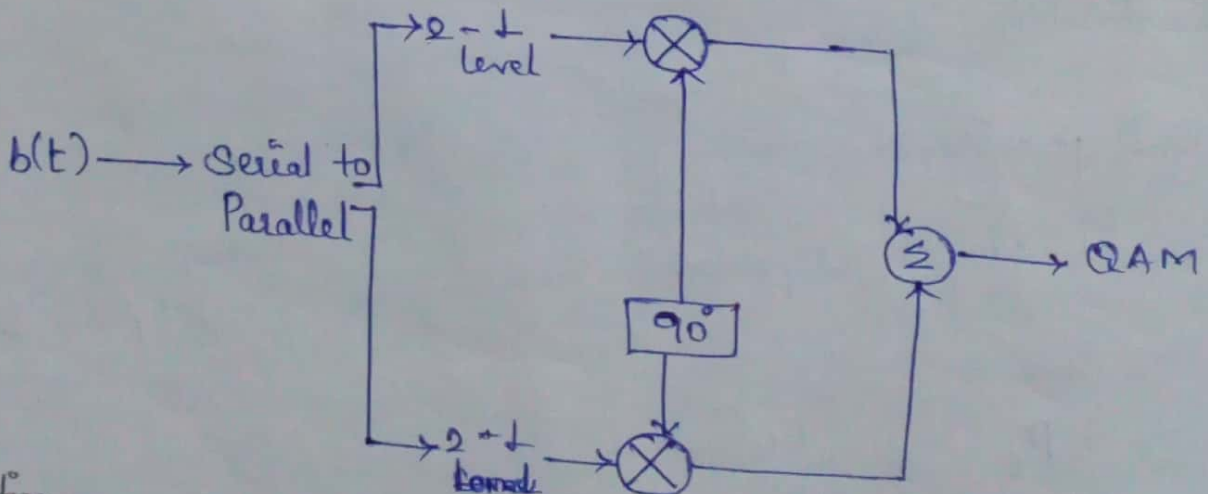
$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \quad \text{and} \quad \phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$$

Geometrical Representation

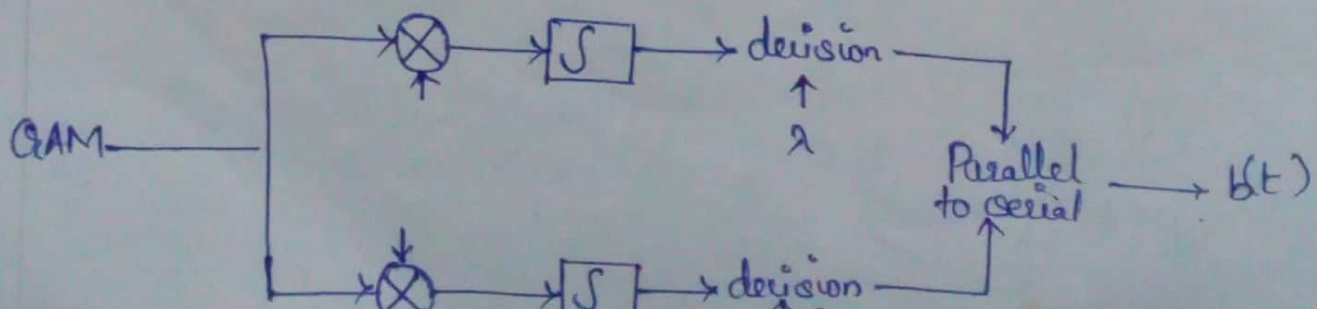
$M=4$



Generation



Detection



Bandwidth $BW = 2f_b / W$.

PSD $S(f) = \frac{P_s T_s}{2} \left[\frac{\sin(\pi f T_s)}{\pi f T_s} \right]^2$

Error Prob.

→ Probability of correct detection

$$P_c = (1 - P_e')^2$$

→ signal constellation

$$P_e' = \left(1 - \frac{1}{L}\right) \text{erfc} \sqrt{\frac{E_b}{N_0}}$$

→ symbol error probability

$$P_e = 2 \left(1 - \frac{1}{\sqrt{M}}\right) \text{erfc} \sqrt{\frac{E_0}{N_0}}$$

Transmitted energy.

$$E_{av} = 2 \left[\frac{2E_0}{L} \sum_{i=1}^{L/2} (2i-1)^2 \right]$$

$$E_{av} = \frac{2(M-1)E_0}{3}$$

Advantages.

- better noise immunity
- Easy to design

Disadvantages.

- high error probability
- complex.

10) Explain DPSK in detail (or) Write a short note on DPSK.

DPSK (Differential Phase Shift Keying).

→ Non coherent version of PSK. Eliminates the need for a coherent reference signal at the rx by combining 2 basic operations at the TX.

↳ differential encoding
↳ PSK.

↳ Sym 0 → phase advance the current signal by 180° .

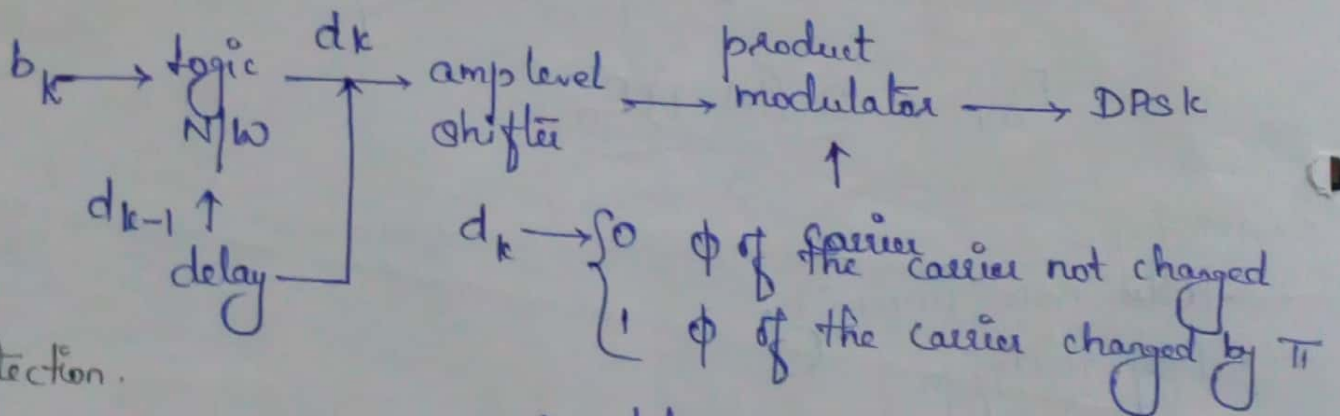
↳ Sym 1 → unchanged the phase.

TX signal Sym 1 → $S_1(t) = \sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t) \quad 0 \leq t \leq 2T_b$

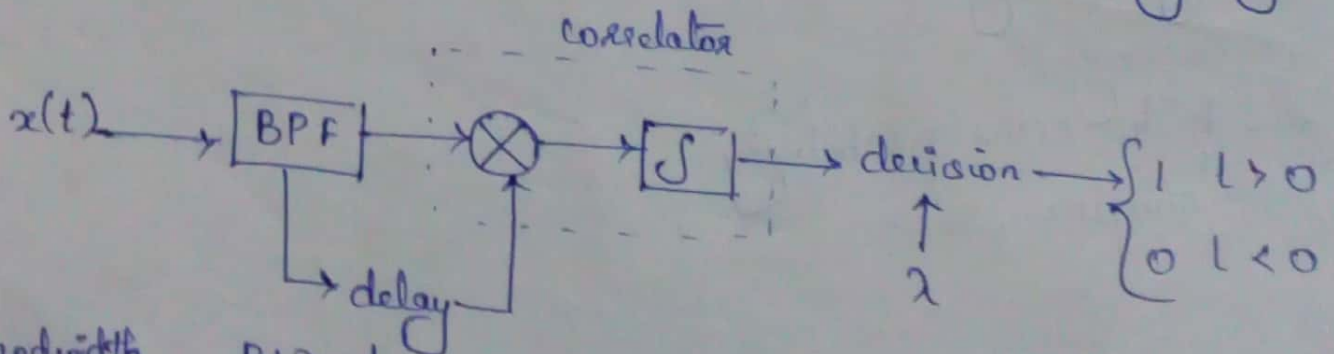
Sym 0 → $S_2(t) = \begin{cases} \sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t) & 0 \leq t \leq T_b \\ \sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t + \pi) & T_b \leq t \leq 2T_b \end{cases}$

$\sqrt{\frac{E_b}{2T_b}} \cos(2\pi f_c t + \pi) \quad T_b \leq t \leq 2T_b$

Generation.



Detection.



Bandwidth

$BW = f_b$

Error Probability $P_e = \frac{1}{2} \exp\left(-\frac{E_b}{N_0}\right)$